All working to be shown clearly.
To be completed and brought to
first Maths lesson in September.

DAVENANT FOUNDATION SCHOOL

Progressing into A level Mathematics

A Equations with brackets

1 Solve the following equations, checking that your answers are correct.

(a) $2-x=3$

(b) $4+2z=-10$

(c) $1-3z=15+4z$

(d) $2=4k-3+k$

(e) $-3u+2=2+u$

(f) $2v-7=2+v$

2 Find the solution of each of the following equations. In each case, check your answers.

(a) $2(w-3)=10$

(b) $5(x-1)=2(x+2)$

(c) $4-2y=3(1-y)$

(d) $4c+3(2-3c)=16$

(e) $5(2-t)=2(t-1)+4(t+3)$

(f) $2(h+1)-4(h-1)=-2+3(h-4)$

B Basic Fractions

1 Carry out the following multiplications and divisions, giving your answers as fractions in their lowest terms.

(a) $\frac{2}{3}\times\frac{5}{4}$

(b) $\frac{5}{6}\times\frac{2}{3}$

(c) $\frac{3}{7}\times\frac{14}{15}$

(d) $\frac{4}{7}\div\frac{3}{14}$

(e) $\frac{8}{9}\div\frac{16}{27}$

(f) $\frac{18}{35}\div\frac{15}{28}$

(g) $2\times\frac{3}{4}$

(h) $1\div\frac{3}{4}$

2 Carry out the following calculations, giving your answers as fractions in their lowest terms.

(a) $\frac{3}{5}+\frac{1}{2}$

(b) $\frac{3}{4}-\frac{4}{7}$

(c) $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}$

(d) $\frac{2}{3}+\frac{3}{4}-\frac{5}{6}$

(e) $\frac{2}{5}+\frac{3}{4}-\frac{3}{10}$

(f) $\frac{8}{7}-\frac{3}{4}-\frac{1}{3}$

(g) $\frac{1}{2}+\frac{1}{4}-\frac{1}{8}$

(h) $\frac{1}{2}\times\frac{1}{4}-\frac{1}{8}$

C Basic Algebraic Fractions

1 Carry out the following multiplications and divisions, giving your answers as fractions in their lowest terms.

(a) $\frac{5}{6}\times\frac{2}{3}$

(b) $\frac{2}{xy}\times\frac{x}{y^2}$

(c) $\frac{3}{y}\times\frac{2}{x}$

(d) $\frac{x^2}{3}\times\frac{6}{x}$

(e) $\frac{8}{9}\div\frac{16}{27}$

(f) $\frac{x}{2}\div\frac{x^2}{4}$

(g) $\frac{2}{5}a^2\div\frac{3}{5}a$

(h) $\frac{12x}{7y}\div\frac{3x^2}{14y^2}$

2 Carry out the following calculations, giving your answers as fractions in their lowest terms.

(a) $\frac{5}{6}+\frac{2}{3}$

(b) $\frac{a}{2}+\frac{b}{4}$

(c) $\frac{1}{3x}+\frac{5}{6x}$

(d) $\frac{3p}{q}-\frac{2p}{3q}$

(e) $\frac{1}{n}+\frac{n}{2}$

(f) $\frac{3p}{q}+\frac{q}{p}$

(g) $4-\frac{5}{x}$

(h) $\frac{3x}{2}-\frac{1}{5x}$

3 Carry out the following calculations, giving your answers as fractions in their lowest terms.

(a) $\frac{1}{2}+\frac{1}{6}-\frac{1}{4}$

(b) $\frac{x}{4}-\frac{x}{2}+\frac{x}{3}$

(c) $\frac{1}{n}+\frac{2}{n}+\frac{3}{n}$

(d) $\frac{p}{2}\times\frac{q}{4}$

(e) $\frac{3}{4}a\div\frac{2}{3}a$

(f) $\frac{1}{2}\times\frac{1}{5}+\frac{1}{3}$

(g) $\frac{x}{4}\times\frac{6}{x^2}-\frac{1}{x}$

(h) $\frac{a}{3}+\frac{b}{2}\times\frac{3a}{b}$

D Indices

1 Simplify the following.

(a) $\left(2^2 \times 3 \times 5^2\right) \times \left(2 \times 3^2 \times 5^3\right)$

(b) $\left(2^2 \times 3 \times 5^2\right) \div \left(2 \times 3^2 \times 5^3\right)$

(c) $\dfrac{2x}{3y} \times \dfrac{6y}{z^2} \times \dfrac{z^3}{2x^3}$

(d) $\left(\dfrac{2a}{3b}\right)^2 \times \left(\dfrac{3b}{c}\right)^3$,

(e) $\left(\dfrac{3r^2}{2s}\right)^3 \div \left(\dfrac{3s}{4t}\right)^2$

(f) $\left(\dfrac{2l}{3m}\right)^3 \times \left(\dfrac{mn}{2l}\right)^2 \div \left(\dfrac{6lm}{n^3}\right)^2$

E Equations with Brackets and Fractions

1 Solve the following equations, checking that your answers are correct.

(a) $2 - x = 3$

(b) $4 + 2z = -10$

(c) $1 - 3z = 15 + 4z$

(d) $2 = 4k - 3 + k$

(e) $-3u + 2 = 2 + u$

(f) $2v - 7 = 2 + v$

2 Find the solution of each of the following equations. In each case, check your answers.

(a) $2(w - 3) = 10$

(b) $5(x - 1) = 2(x + 2)$

(c) $4 - 2y = 3(1 - y)$

(d) $4c + 3(2 - 3c) = 16$

(e) $5(2 - t) = 2(t - 1) + 4(t + 3)$

(f) $2(h + 1) - 4(h - 1) = -2 + 3(h - 4)$

3 Find the solution of each of the following equations.

(a) $\dfrac{1 - 2x}{3} - \dfrac{1 + 3x}{4} = 10$

(b) $\dfrac{x}{6} - \dfrac{x - 2}{3} = 4$

(c) $\frac{2}{3}(x + 1) - \frac{3}{5}(2x - 3) = \frac{1}{3}$

(d) $\frac{4}{7}(2x - 1) - \frac{1}{2}(x - 5) = 2$

(e) $\dfrac{1 + 2x}{5} - \dfrac{5 + 3x}{4} = \dfrac{x - 4}{2} - 5$

(f) $\frac{2}{3}(x + 4) - \frac{3}{4}(2x + 5) = \frac{1}{12}(x - 2)$

F Basic Factorising and Expanding

1 Factorise the following expressions.

(a) $ax + 2x$

(b) $pq + p^2$

(c) $2p + 3q + 6 + pq$

(d) $4z - 3t + 12 - tz$

(e) $2h^2 + 3h - 2hk - 3k$

(f) $ax + ay - az$

2 Expand the following brackets, simplifying your answer where possible.

(a) $(a + 2)(c - 2)$

(b) $(2p + 1)(p - 1)$

(c) $(x - y)(x - 2z)$

(d) $(x + 3y)(x - 3y)$

(e) $(y - 3z)^2$

(f) $(2p + 3q)^2$

G Basic Change of Subject

1 Solve each of the following equations for x.

(a) $ax + bx = x(a - b) + c$

(b) $s = \frac{1}{2}n(x + y)$

(c) $b(x + a) = ab$

(d) $x(b + c) - a = b$

(e) $\dfrac{x}{a} + \dfrac{y}{b} = \dfrac{xy}{ab}$

(f) $\dfrac{x^3}{a^3} + \dfrac{y^3}{b^3} = 1$

2 In each case, make the letter given at the end the subject of the formula.

(a) $s = \dfrac{a}{1 - r}$, r

(b) $A = 2\pi r(r + h)$, h

(c) $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f}$, f

(d) $h = \frac{1}{2}gt^2$, t

(e) $v^2 = w^2(a^2 - x^2)$, x

(f) $V = \frac{1}{3}\sqrt{\dfrac{s^3}{8\pi}}$, s

H Factorising Quadratics

1 Factorise each of the following quadratics.

(a) x^2+3x+2 (b) x^2+4x+3 (c) x^2+4x+4

(d) y^2+9y+8 (e) $x^2+7x+10$ (f) x^2+6x+9

(g) $x^2+11x+24$ (h) $p^2+9p+20$ (i) $x^2+9x+18$

(j) $k^2+22k+21$ (k) $x^2+11x+30$ (l) $x^2+19x+60$

2 Factorise each of the following quadratics.

(a) x^2-4x+3 (b) a^2-5a+4 (c) x^2-7x+6

(d) x^2-9x+8 (e) x^2-2x+1 (f) $l^2-7l+12$

(g) $z^2-13z+22$ (h) $x^2-10x+25$ (i) $x^2-11x+28$

(j) $x^2-12x+32$ (k) $q^2-12q+35$ (l) $x^2-16x+60$

3 Factorise each of the following quadratics.

(a) x^2+x-2 (b) x^2-x-2 (c) c^2-3c-4

(d) x^2+x-6 (e) d^2-2d-8 (f) x^2+7x-8

(g) $h^2+9h-22$ (h) x^2-x-12 (i) $x^2+5x-24$

(j) $x^2-3x-18$ (k) $x^2+10x-56$ (l) $n^2-7n-60$

4 Factorise each of the following quadratics.

(a) $2x^2+3x+1$ (b) $2x^2-5x+2$ (c) $3x^2+5x+2$

(d) $2x^2-7x+5$ (e) $3x^2-8x+4$ (f) $6x^2+5x+1$

(g) $9x^2-6x+1$ (h) $9x^2+12x+4$ (i) $4x^2-12x+5$

(j) $6x^2-13x+6$ (k) $3x^2+14x+8$ (l) $60x^2+19x+1$

5 Factorise each of the following quadratics.

(a) $2x^2-x-1$ (b) $3x^2+2x-1$ (c) $6x^2-x-1$

(d) $12x^2+x-1$ (e) $2x^2+3x-2$ (f) $2x^2-x-3$

(g) $4x^2-7x-2$ (h) $3x^2-4x-4$ (i) $8x^2-15x-2$

(j) $6x^2+5x-6$ (k) $4x^2-8x-5$ (l) $6x^2-11x-10$

6 Find the factors of each of the following quadratics.

(a) x^2-1 (b) x^2-81 (c) $4x^2-9$

(d) $9x^2-1$ (e) $4-25d^2$ (f) $8z^2-32$

7 Find the factors of each of the following quadratics.

(a) $3x^2-27$ (b) $50-2y^2$ (c) $100-4z^2$

(d) $(x+1)^2-4x^2$ (e) $(2x+1)^2-x^2$ (f) $(2x+1)^2-(x-3)^2$

I Solving Quadratic Equations

1 Use the quadratic equation formula to solve the following quadratic equations, which all have solutions that are whole numbers or fractions.

(a) $x^2+7x+10=0$ (b) $x^2-9x+20=0$ (c) $x^2-3x-18=0$

(d) $2x^2-7x+5=0$ (e) $3p^2+4p-4=0$ (f) $3+4q-4q^2=0$

(g) $9x^2-24x+16=0$ (h) $x^2=6x+27$ (i) $3x^2=16-13x$

2 Use the quadratic equation formula to solve the following quadratic equations, giving your answers correct to 3 significant figures.

(a) $x^2+7x+9=0$ (b) $x^2-9x+9=0$ (c) $x^2=3x+5$

(d) $2x^2-8x-5=0$ (e) $3y^2+4y-6=0$ (f) $3x^2-2x-7=0$

(g) $5x^2=6+3x$ (h) $6+3x=8x^2$ (i) $4x^2-x-7=0$

J Basic Simultaneous Equations

1 Solve the following pairs of simultaneous equations.

(a) $\begin{aligned} x-y &= 8 \\ x+y &= 13 \end{aligned}$

(b) $\begin{aligned} x+2y &= 17 \\ 2x+3y &= 16 \end{aligned}$

(c) $\begin{aligned} 5x-8y &= 4 \\ 2x+3y &= -17 \end{aligned}$

(d) $\begin{aligned} 2x-4y &= 6 \\ 2x+4y &= 6 \end{aligned}$

(e) $\begin{aligned} 5x &= 1-4y \\ 5y &= 1-6x \end{aligned}$

(f) $\begin{aligned} 0.2x-\tfrac{1}{2}y &= -1 \\ 0.1x+\tfrac{1}{5}y &= 4 \end{aligned}$

K Sine and Cosine Rules

1 Find the areas of the following triangles, giving your answers correct to 3 significant figures.

(a) $x = 43$ cm, $y = 65$ cm, $Z = 12°$

(b) $a = 11.3$ cm, $b = 13.5$ cm, $C = 130°$

2 In each of the following triangles find the lengths of the sides, giving your answers to 3 significant figures.

(a) $p = 9.2$ cm, $R = 130°$, $P = 32°$: find q and r.

(b) $a = 15.3$ cm, $A = 14°$, $C = 130°$: find b and c.

3 In each of the following triangles find the required angles, giving your answers correct to 1 decimal place.

(a) $x = 8.5$ cm, $y = 7.5$ cm, $X = 82°$: find Y, Z.

(b) $a = 7.3$ cm, $c = 5.3$ cm, $C = 10°$: find A, B.

4 Find the length of the third side of the triangle in the following two cases.

(a) $x = 3.8$ cm, $y = 5.2$ cm, $Z = 67.3°$

(b) $l = 14.2$ cm, $m = 13.3$ cm, $N = 134.2°$

5 Find all the angles of the triangle whose sides are

(a) 4.2 cm, 5.3 cm, 6.4 cm,

(b) 5.3 cm, 5.2 cm, 9.9 cm.

L Simultaneous Equations – Intersection of Line and Curve

Find the coordinates of the points of intersection of the given straight lines with the given curves. (In other words, solve the following pairs of simultaneous equations.)

1. $y = x+1$ $x^2 + y^2 = 25$

2. $y = 7-x$ $x^2 + y^2 = 25$

3. $y = x-3$ $y = x^2 - 3x - 8$

4. $y = 2x+1$ $y = x^2 - x + 3$

5. $y = 2x-2$ $y = x^2 - 5$

6. $y = 2x-12$ $x^2 + 4xy - 3y^2 = -27$

Progressing into A level Mathematics

A Equations with brackets

1 (a) -1 (b) -7 (c) -2
  (d) 1 (e) 0 (f) 9

2 (a) 8 (b) 3 (c) -1
  (d) -2 (e) 0 (f) 4

B Basic Fractions

1 (a) $\frac{5}{6}$ (b) $\frac{5}{9}$ (c) $\frac{2}{5}$ (d) $\frac{8}{3}$
  (e) $\frac{3}{2}$ (f) $\frac{24}{25}$ (g) $\frac{3}{2}$ (h) $\frac{4}{3}$

2 (a) $\frac{11}{10}$ (b) $\frac{5}{28}$ (c) $\frac{9}{4}$ (d) $\frac{7}{12}$
  (e) $\frac{17}{20}$ (f) $\frac{5}{84}$ (g) $\frac{5}{8}$ (h) 0

C Basic Algebraic Fractions

1 (a) $\frac{5}{9}$ (b) $\frac{2}{y^3}$ (c) $\frac{6}{xy}$
  (d) $2x$ (e) $\frac{3}{2}$ (f) $\frac{2}{x}$
  (g) $\frac{2a}{3}$ (h) $\frac{8y}{x}$

2 (a) $\frac{3}{2}$ (b) $\frac{2a+b}{4}$ (c) $\frac{7}{6x}$
  (d) $\frac{7p}{3q}$ (e) $\frac{2+n^2}{2n}$ (f) $\frac{3p^2+q^2}{pq}$
  (g) $\frac{4x-5}{x}$ (h) $\frac{15x^2-2}{10x}$

3 (a) $\frac{5}{12}$ (b) $\frac{1}{12}x$ (c) $\frac{6}{n}$
  (d) $\frac{1}{8}pq$ (e) $\frac{9}{8}$ (f) $\frac{13}{30}$
  (g) $\frac{1}{2x}$ (h) $\frac{11a}{6}$

D Indices

1 (a) $2^3 \times 3^3 \times 5^5 = 675\,000$
  (b) $\frac{2}{3\times5} = \frac{2}{15}$ (c) $\frac{2z}{x^2}$
  (d) $\frac{12a^2 b}{c^3}$ (e) $\frac{6r^6 t^2}{s^5}$
  (f) $\frac{n^8}{486lm^3}$

E Equations with Brackets and Fractions

1 (a) -1 (b) -7 (c) -2
  (d) 1 (e) 0 (f) 9

2 (a) 8 (b) 3 (c) -1
  (d) -2 (e) 0 (f) 4

3 (a) -7 (b) -20 (c) 4
  (d) $\frac{1}{9}$ (e) 7 (f) -1

F Basic Factorising and Expanding

1 (a) $x(a+2)$ (b) $p(q+p)$
  (c) $(q+2)(3+p)$ (d) $(4-t)(z+3)$
  (e) $(2h+3)(h-k)$ (f) $a(x+y-z)$

2 (a) $ac-2a+2c-4$ (b) $2p^2-p-1$
  (c) $x^2-2xz-xy+2yz$ (d) x^2-9y^2
  (e) $y^2-6yz+9z^2$
  (f) $4p^2+12pq+9q^2$

G Basic Change of Subject

1 (a) $x = \dfrac{c}{2b}$ (b) $x = \dfrac{2s-ny}{n}$

  (c) $x = 0$ (d) $x = \dfrac{a+b}{b+c}$

  (e) $x = \dfrac{ay}{y-b}$ (f) $x = \sqrt[3]{\dfrac{a^3b^3 - a^3y^3}{b^3}}$

2 (a) $r = \dfrac{s-a}{s}$ (b) $h = \dfrac{A-2\pi r^2}{2\pi r}$

  (c) $f = \dfrac{uv}{u+v}$ (d) $t = \pm\sqrt{\dfrac{2h}{g}}$

  (e) $x = \pm\sqrt{\dfrac{w^2a^2 - v^2}{w^2}}$

  (f) $s = \sqrt[3]{72\pi V^2}$

H Factorising Quadratics

1 (a) $(x+1)(x+2)$ (b) $(x+1)(x+3)$
  (c) $(x+2)^2$ (d) $(y+1)(y+8)$
  (e) $(x+2)(x+5)$ (f) $(x+3)^2$
  (g) $(x+3)(x+8)$ (h) $(p+4)(p+5)$
  (i) $(x+3)(x+6)$ (j) $(k+1)(k+21)$
  (k) $(x+5)(x+6)$ (l) $(x+4)(x+15)$

2 (a) $(x-1)(x-3)$ (b) $(a-1)(a-4)$
  (c) $(x-1)(x-6)$ (d) $(x-1)(x-8)$
  (e) $(x-1)^2$ (f) $(l-3)(l-4)$
  (g) $(z-2)(z-11)$ (h) $(x-5)^2$
  (i) $(x-4)(x-7)$ (j) $(x-4)(x-8)$
  (k) $(q-5)(q-7)$ (l) $(x-6)(x-10)$

3 (a) $(x-1)(x+2)$ (b) $(x-2)(x+1)$
  (c) $(c-4)(c+1)$ (d) $(x-2)(x+3)$

  (e) $(d-4)(d+2)$ (f) $(x-1)(x+8)$
  (g) $(h-2)(h+11)$ (h) $(x-4)(x+3)$
  (i) $(x-3)(x+8)$ (j) $(x-6)(x+3)$
  (k) $(x-4)(x+14)$ (l) $(n-12)(n+5)$

4 (a) $(x+1)(2x+1)$ (b) $(x-2)(2x-1)$
  (c) $(x+1)(3x+2)$ (d) $(x-1)(2x-5)$
  (e) $(x-2)(3x-2)$ (f) $(2x+1)(3x+1)$
  (g) $(3x-1)^2$ (h) $(3x+2)^2$
  (i) $(2x-1)(2x-5)$ (j) $(2x-3)(3x-2)$
  (k) $(3x+2)(x+4)$ (l) $(4x+1)(15x+1)$

5 (a) $(2x+1)(x-1)$ (b) $(3x-1)(x+1)$
  (c) $(3x+1)(2x-1)$ (d) $(4x-1)(3x+1)$
  (e) $(2x-1)(x+2)$ (f) $(2x-3)(x+1)$
  (g) $(4x+1)(x-2)$ (h) $(3x+2)(x-2)$
  (i) $(8x+1)(x-2)$ (j) $(2x+3)(3x-2)$
  (k) $(2x-5)(2x+1)$ (l) $(3x+2)(2x-5)$

6 (a) $(x+1)(x-1)$ (b) $(x+9)(x-9)$
  (c) $(2x+3)(2x-3)$ (d) $(3x+1)(3x-1)$
  (e) $(2+5d)(2-5d)$ (f) $8(z+2)(z-2)$

7 (a) $3(x+3)(x-3)$ (b) $2(5+y)(5-y)$
  (c) $4(5+z)(5-z)$ (d) $(1-x)(1+3x)$
  (e) $(x+1)(3x+1)$ (f) $(x+4)(3x-2)$

I Solving Quadratic Equations

1 (a) $-2, -5$ (b) $4, 5$ (c) $-3, 6$
  (d) $1, \frac{5}{2}$ (e) $-2, \frac{2}{3}$ (f) $-\frac{1}{2}, \frac{3}{2}$
  (g) $\frac{4}{3}$ (h) $-3, 9$ (i) $-\frac{16}{3}, 1$

2 (a) $-5.30, -1.70$ (b) $1.15, 7.85$
  (c) $-1.19, 4.19$ (d) $-0.550, 4.55$
  (e) $-2.23, 0.897$ (f) $-1.23, 1.90$
  (g) $-0.836, 1.44$ (h) $-0.699, 1.07$
  (i) $-1.20, 1.45$

J Basic Simultaneous Equations

1 (a) $x = \frac{21}{2}, y = \frac{5}{2}$ (b) $x = -19, y = 18$
  (c) $x = -4, y = -3$ (d) $x = 3, y = 0$
  (e) $x = 1, y = -1$ (f) $x = 20, y = 10$

K Sine and Cosine Rules

1 (a) $291\,\text{cm}^2$ (b) $58.4\,\text{cm}^2$

2 (a) 5.36 cm, 13.3 cm
  (b) 37.2 cm, 48.4 cm

3 (a) $Y = 60.9°$, $Z = 37.1°$
  (b) $A = 13.8°$, $B = 156.2°$ or $A = 166.2°$, $B = 3.8°$

4 (a) 5.12 cm (b) 25.3 cm

5 (a) $83.8°, 40.7°, 55.4°$
  (b) $141.1°, 19.7°, 19.3°$

L Simultaneous Equations – Intersection of Line and Curve

1. $(3, 4)$ $(-4, -3)$ 3. $(5, 2)$ $(-1, -4)$ 5. $(3, 4)$ $(-1, -4)$

2. $(3, 4)$ $(4, 3)$ 4. $(2, 5)$ $(1, 3)$ 6. $(5, -2)$ $(27, 42)$